



## Seminario di Analisi Matematica

Mercoledì 23 marzo 2022, ore 15:00  
Dipartimento di Matematica e Fisica  
Aula 211, Palazzina C, L.go S.L. Murialdo 1

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### **ON THE GENERICITY OF EFFECTIVELY STABLE INTEGRABLE HAMILTONIAN SYSTEMS AND ON THEIR ALGEBRAIC PROPERTIES.**

#### Abstract

Hamiltonian systems constitute an important class of dynamical systems. Those hamiltonian systems which are integrable in the sense of Arnold-Liouville possess an important property: their solutions can be written explicitly and the phase space is foliated by invariant tori carrying global quasi-periodic orbits. This kind of systems are exceptional but in applications it is not rare to see systems which are perturbations of integrable ones. A natural question is then to determine whether the stability of solutions is preserved for this latter type of systems. Kolmogorov-Arnold-Moser theory assures that, under generic hypotheses, a Cantor set of positive Lebesgue measure of invariant tori carrying quasi-periodic motions persists under a sufficiently small perturbation. On the other hand, instabilities may appear in the complementary of this set (Arnold diffusion). Moreover, a Theorem due to Nekhoroshev (1971-1977) shows that the solutions of a sufficiently regular integrable system verifying a transversality property known as steepness are stable over a long time under the effect of a suitably small perturbation. Nekhoroshev also showed (1973) that the steepness property is generic, both in measure and topologic sense, in the space of jets (Taylor polynomials) of sufficiently smooth functions. However, the proof of this result kept being poorly understood up to now and, surprisingly, the paper in which it is contained is hardly known, whereas the rest of the theory has been widely studied over the decades. Moreover, the definition of steepness is not constructive and no general rule to establish whether a given function is steep or not existed up to now, thus entailing a major problem in applications.

In this seminar, I will start by explaining the main ideas behind Nekhoroshev's proof of the genericity of steepness by making use of a more modern language. Indeed, the proof strongly relies on arguments complex analysis and real algebraic geometry: the latter was much less developed than nowadays at the time that Nekhoroshev was writing, so that many passages appear to be quite obscure in the original article. Moreover, an important result of real algebraic geometry was buried in the proof and seems to have been proved again by Roytwarf and Yomdin in 1997 by making use of different arguments (and generalized in many directions by subsequent works of many authors). Finally, I will show how a deep understanding of the genericity of steepness allows to determine explicit algebraic criteria in the space of jets which make it possible to establish whether a given function is steep or not.

Reference: N. N. Nekhoroshev, Stable lower estimates for smooth mappings and for gradients of smooth function, Mathematics of the USSR-Sbornik, 1973, vol. 90 (132), no. 3, pp.432-478